Journal of Lie Theory 23 (2013) 669–689

A. S. Gordienko
Dept. of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NL, Canada A1C 5S7
gordienko.a.s@gmail.com

The Structure of H-(co)module Lie algebras

Let L be a finite dimensional Lie algebra over a field of characteristic 0. Then by the original Levi theorem, $L = B \oplus R$ where R is the solvable radical and B is some maximal semisimple subalgebra. We prove that if L is an H-(co)module algebra for a finite dimensional (co)semisimple Hopf algebra H, then R is H-(co)invariant and B can be chosen to be H-(co)invariant too. Moreover, the nilpotent radical N of L is H-(co)invariant and there exists an H-sub(co)module $S \subseteq R$ such that $R = S \oplus N$ and $[B, S] = 0$. In addition, the H-(co)invariant analog of the Weyl theorem is proved. In fact, under certain conditions, these results hold for an H-comodule Lie algebra L, even if H is infinite dimensional. In particular, if L is a Lie algebra graded by an arbitrary group G, then B can be chosen to be graded, and if L is a Lie algebra with a rational action of a reductive affine algebraic group G by automorphisms, then B can be chosen to be G-invariant. Also we prove that every finite dimensional semisimple H-(co)module Lie algebra over a field of characteristic 0 is a direct sum of its minimal H-(co)invariant ideals.

Keywords: Lie algebra, stability, Levi decomposition, radical, grading, Hopf algebra, Hopf algebra action, H-module algebra, H-comodule algebra.

MSC: 17B05; 17B40, 17B55, 17B70, 16T05, 14L17