Induced $\ast$-Representations and $C^*$-Envelopes of Some Quantum $\ast$-Algebras

We consider three quantum algebras: the $q$-oscillator algebra, the Podleś sphere and the $q$-deformed enveloping algebra of $su(2)$. To each of these $\ast$-algebras we associate a certain partial dynamical system and perform the “Mackey analysis” of $\ast$-representations developed by Yu. Savchuk and K. Schmüdgen [“Unbounded induced representations of $\ast$-algebras”, Algebr. Represent. Theory, DOI: 10.1007/s10468-011-9310-6]. As a result we get the description of “standard” irreducible $\ast$-representations. Further, for each of these examples we show the existence of a “$C^*$-envelope” which is canonically isomorphic to the covariance $C^*$-algebra of the partial dynamical system. Finally, for the $q$-oscillator algebra and the $q$-deformed $U(su(2))$ we show the existence of “bad” representations.

Keywords: Induced representations, group graded algebras, well-behaved representations, partial action of a group, Mackey analysis, $C^*$-envelope, $q$-deformed enveloping algebra, Podles sphere, $q$-oscillator.

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