

K. Coulembier

Dept. of Mathematical Analysis, Ghent University, Krijgslaan 281, 9000 Gent, Belgium
and: School of Mathematics and Statistics, University of Sydney, Sydney, Australia
coulembier@cage.ugent.be

The Orthosymplectic Superalgebra in Harmonic Analysis

We introduce the orthosymplectic superalgebra $\mathfrak{osp}(m|2n)$ as the algebra of Killing vector fields on Riemannian superspace $\mathbb{R}^{m|2n}$ which stabilize the origin. The Laplace operator and norm squared on $\mathbb{R}^{m|2n}$, which generate \mathfrak{sl}_2 , are orthosymplectically invariant, and we therefore obtain the Howe dual pair

$$(\mathfrak{osp}(m|2n)(m|2n), \mathfrak{sl}_2) .$$

We study the $\mathfrak{osp}(m|2n)$ -representation structure of the kernel of the Laplace operator. This also yields the decomposition of the supersymmetric tensor powers of the fundamental $\mathfrak{osp}(m|2n)$ -representation under the action of $\mathfrak{sl}_2 \times \mathfrak{osp}(m|2n)$. As a side result we obtain information about the irreducible $\mathfrak{osp}(m|2n)$ -representations $L_{(k,0,\dots,0)}^{m|2n}$. In particular we find branching rules with respect to $\mathfrak{osp}(m|2n)(m-1|2n)$. We also prove that integration over the supersphere is uniquely defined by its orthosymplectic invariance.

Keywords: Howe dual pair, orthosymplectic superalgebra, not completely reducible representations, supersymmetric tensor product.

MSC: 17B10, 58C50, 17B15