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Howe Duality for the Metaplectic Group Acting on Symplectic Spinor Valued Forms

Let $\mathbb{S}$ denote the oscillatory module over the complex symplectic Lie algebra $\mathfrak{g} = \mathfrak{sp}(\mathbb{V}^{\mathbb{C}}, \omega)$. Consider the $\mathfrak{g}$-module $\mathbb{W} = \bigwedge^{\bullet}(\mathbb{V}^*)^{\mathbb{C}} \otimes \mathbb{S}$ of forms with values in the oscillatory module. We prove that the associative commutant algebra $\mathrm{End}_{\mathfrak{g}}(\mathbb{W})$ is generated by the image of a certain representation of the ortho-symplectic Lie super algebra $\mathfrak{osp}(1|2)$ and two distinguished projection operators. The space $\mathbb{W}$ is then decomposed with respect to the joint action of $\mathfrak{g}$ and $\mathfrak{osp}(1|2)$. This establishes a Howe type duality for $\mathfrak{sp}(\mathbb{V}^{\mathbb{C}}, \omega)$ acting on $\mathbb{W}$.

Keywords: Howe duality, symplectic spinors, Segal-Shale-Weil representation, Kostant spinor.

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