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Picard Groups of Siegel Modular 3-Folds and θ-Liftings

We show that the Humbert surfaces rationally generate the Picard groups of Siegel modular threefolds. This involves three ingredients: (1) R. Weissauer's determination of these Picard groups in terms of theta lifting from cusp forms of weight $5/2$ on $\tilde{\mathrm{SL}}_2(\mathbb{R})$ to automorphic forms on $\mathrm{Sp}_4(\mathbb{R})$. (2) The theory of special cycles due to Kudla/Millson and Tong/Wang relating cohomology defined by automorphic forms to that defined by certain geometric cycles. (3) Results of R. Howe about the structure of the oscillator representation in this situation.

Keywords: Siegel modular threefold, Picard group, theta lifting.

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