The Tame Algebra

The tame subgroup $I_t$ of the Iwahori subgroup $I$ and the tame Hecke algebra $H_t = C_c(I_t \backslash G/I_t)$ are introduced. It is shown that the tame algebra has a presentation by means of generators and relations, similar to that of the Iwahori-Hecke algebra $H = C_c(I \backslash G/I)$. From this it is deduced that each of the generators of the tame algebra is invertible. This has an application concerning an irreducible admissible representation $\pi$ of an unramified reductive $p$-adic group $G$: $\pi$ has a nonzero vector fixed by the tame group, and the Iwahori subgroup $I$ acts on this vector by a character $\chi$, iff $\pi$ is a constituent of the representation induced from a character of the minimal parabolic subgroup, denoted $\chi_A$, which extends $\chi$. The proof is an extension to the tame context of an unpublished argument of Bernstein, which he used to prove the following. An irreducible admissible representation $\pi$ of a quasisplit reductive $p$-adic group has a nonzero Iwahori-fixed vector iff it is a constituent of a representation induced from an unramified character of the minimal parabolic subgroup. The invertibility of each generator of $H_t$ is finally used to give a Bernstein-type presentation of $H_t$, also by means of generators and relations, as an extension of an algebra with generators indexed by the finite Weyl group, by a finite index maximal commutative subalgebra, reflecting more naturally the structure of $G$ and its maximally split torus.

Keywords: Tame algebra, Iwahori-Hecke Algebra, induced representation.

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