Invariant Berezin Integration on Homogeneous Supermanifolds

Let $G$ be a Lie supergroup and $H$ a closed subsupergroup. We study the unimodularity of the homogeneous supermanifold $G/H$, i.e. the existence of $G$-invariant sections of its Berezinian line bundle. To that end, we express this line bundle as a $G$-equivariant associated bundle of the principal $H$-bundle $G \to G/H$. We also study the fibre integration of Berezinians on oriented fibre bundles. As an application, we prove a formula of ‘Fubini’ type:

$$\int_G f = (-1)^{\dim h_1 \cdot \dim g/h} \int_{G/H} \int_H f, \text{ for all } f \in \Gamma_c(G, \mathcal{O}_G).$$

Moreover, we derive analogues of integral formulae for the transformation under local isomorphisms $G/H \to S/T$, and under the products of Lie subsupergroups $M \cdot H \subset U$. The classical counterparts of these formulae have numerous applications in harmonic analysis.

**Keywords**: Supermanifold, Lie supergroup, homogeneous superspace, Berezin integral, invariant Berezinian form, unimodularity, Fubini formula, fibre integration.

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