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A Combinatorial Basis for the Free Lie Algebra of the Labelled Rooted Trees

The pre-Lie operad is an operad structure on the species $\mathcal{T}$ of labelled rooted trees. A result of F. Chapoton shows that the pre-Lie operad is a free twisted Lie algebra over a field of characteristic zero, that is $\mathcal{T} = \mathcal{L}ie \circ \mathcal{F}$ for some species $\mathcal{F}$. Indeed Chapoton proves that any section of the indecomposables of the pre-Lie operad, viewed as a twisted Lie algebra, gives such a species $\mathcal{F}$.
In this paper, we first construct an explicit vector space basis of $\mathcal{F}[S]$ when S is a linearly ordered set. We deduce the associated explicit species $\mathcal{F}$, solution to the equation $\mathcal{T} = \mathcal{L}ie \circ \mathcal{F}$. As a corollary the graded vector space $(\mathcal{F}[\{1, \ldots, n\}])_{n\in\mathbb{N}}$ forms a sub non-symmetric operad of the pre-Lie operad $\mathcal{T}$.

Keywords: Free Lie algebra, rooted tree, pre-Lie operad, Lyndon word.

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