Given a Lie group $G$ with finitely many components and a compact Lie group $A$ which acts on $G$ by automorphisms, we prove that there always exists an $A$-invariant maximal compact subgroup $K$ of $G$, and that for every such $K$, the natural map $H^1(A, K) \to H^1(A, G)$ is bijective. This generalizes a classical result of Serre and a recent result of the first and third named authors of the current paper.

Keywords: Nonabelian cohomology, compact Lie group, maximal compact subgroup.

MSC: 20J06, 22E15, 57S15