The Representation Aspect of the Generalized Hydrogen Atoms

Let $D \geq 1$ be an integer. In the Enright-Howe-Wallach classification list of the unitary highest weight modules of $\tilde{\text{Spin}}(2, D + 1)$, the (nontrivial) Wallach representations in Case II, Case III, and the mirror of Case III are special in the sense that they are precisely the ones that can be realized by the Hilbert space of bound states for a generalized hydrogen atom in dimension $D$. It has been shown recently that each of these special Wallach representations can be realized as the space of $L^2$-sections of a canonical hermitian bundle over the punctured $\mathbb{R}^D$. Here a simple algebraic characterization of these special Wallach representations is found.

**Keywords**: Kepler problem, Harish-Chandra modules.

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