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Some Basic Results Concerning G-invariant Riemannian Metrics

we study complete G-invariant Riemannian metrics. Let G be a Lie group and let M be a proper smooth G-manifold. Let α be a smooth G-invariant Riemannian metric of M, and let \tilde{K} be any G-compact subset of M. We show that M admits a complete smooth G-invariant Riemannian metric β such that $\beta | \tilde{K} = \alpha | \tilde{K}$. We also prove the existence of complete real analytic G-invariant Riemannian metrics for proper real analytic G-manifolds. Moreover, we show that for any given smooth (real analytic) G-invariant Riemannian metric there exists a complete smooth (real analytic) G-invariant Riemannian metric conformal to the original Riemannian metric. To prove the real analytic results we need the assumption that G can be embeddded as a closed subgroup of a Lie group which has only finitely many connected components.

Keywords: Lie groups, Riemannian metric, real analytic.

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