An Asymptotic Result on the A-Component in the Iwasawa Decomposition

Let $G$ be a connected noncompact semisimple Lie group. For each $v', v, g \in G$, we prove that

$$\lim_{t \to \infty} [a(v'g^tv)]^{1/t} = s^{-1} \cdot b(g),$$

where $a(g)$ denotes the $a$-component in the Iwasawa decomposition of $g = kan$ and $b(g) \in A_+$ denotes the unique element that is conjugate to the hyperbolic component $h$ in the complete multiplicative Jordan decomposition of $g = ehu$. The element $s$ in the Weyl group of $(G, A)$ is determined by $yv \in G$ (not unique in general) in such a way that $yv \in N^-m_sMAN$, where $yhy^{-1} = b(g)$ and $G = \cup_{s \in W} N^-m_sMAN$ is the Bruhat decomposition of $G$.

**Keywords:** Iwasawa decomposition, complete multiplicative Jordan decomposition, Bruhat decomposition, a-component.

**MSC:** 22E46; 22E30