Operator Kernels for Irreducible Unitary Representations of Solvable Exponential Lie Groups

Let $G$ be a connected, simply connected, exponential solvable Lie group. The irreducible unitary representations of $G$ may be obtained by the Kirillov-Bernat orbit method. Let $\ell \in \mathfrak{g}^*$, $\mathfrak{p}$ a Pukanszky polarization associated to $\ell$, $P = \exp \mathfrak{p}$, $\chi_\ell$ the corresponding character of $P$ and $\pi_\ell = \text{ind}_P^G \chi_\ell$ the associated unitary representation. We show through an example that not all the functions of $C^\infty_c(G/P, G/P, \chi_\ell)$ ($C^\infty$-functions with compact support on $G/P \times G/P$ satisfying a certain covariance condition) are kernel functions of some operator of the form $\pi_\ell(f)$, $f \in L^1(G)$, even if the polarization is well chosen. This contradicts a result of H. Leptin [J. Reine Angew. Math. 494 (1998) 1–34]). But if the polarization $\mathfrak{p}$ is an ideal of $\mathfrak{g}$, then the result of Leptin is true, the corresponding retract from $C^\infty_c(G/P, G/P, \chi_\ell)$ into $L^1(G)$ exists and a construction algorithm of the function $f$ may be indicated.

Keywords: Irreducible unitary representation, kernel of an operator, retract.

MSC: 43A20