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Classification des Structures CR Invariantes pour les Groupes de Lie Compacts

Let $G_0$ be a compact Lie group of dimension $N$ whose Lie algebra is $\mathfrak{g}_0$. The notion of CR structure on a $C^\infty$ manifold is known a long time ago. In this note we are interested by the CR structures on $G_0$ which are invariant by the left action of the group on the tangent bundle and which are of maximal rank. Such a structure is defined by its fibre $\mathfrak{h}$ at the neutral element which is a subalgebra of the complexification $\mathfrak{g}$ of $\mathfrak{g}_0$ whose dimension is the entire part $[N/2]$ of $N/2$ and whose intersection with $\mathfrak{g}_0$ is equal to $\{0\}$. Up to conjugation by the adjoint group of $\mathfrak{g}_0$, these subalgebras are classified. When $N$ is even, there is only one type, type $CR_0$. When $N$ is odd, there are two types, type $CR_0$ and type $CRI$. These types are given in terms of Cartan subalgebras and root systems. In any case, these subalgebras are solvable. Following M. S. Baouendi, L. P. Rothschild and F. Treves ["CR structures with group action and extendability of CR functions", Inventiones Mathematicae 82 (1985) 359–396], we introduce the notion of CR structures which are $G_0$-invariant and invariant by the transverse action of a $G_0$-invariant Lie subgroup. When this group is commutative, we get the notion of $G_0$-rigidity. We then prove, when $N$ is odd, that a $G_0$-invariant CR structure, of maximal rank, is $G_0$-rigid if and only if the fibre of the CR structure at the neutral element, is of type $CR_0$. Following H. Jacobowitz ["The canonical bundle and realizable CR hypersurfaces", Pacific Journal of Mathematics 127 (1987) 91–101], we introduce the canonical fibre bundle $K$ of a $G_0$-invariant CR structure, of maximal rank, when $N$ is odd. We prove that $K$ contains a closed $G_0$-invariant form if and only if the fibre of the CR structure at the neutral element, is of type $CR_0$ or type $CRII$. As for type $CR_0$ and $CRI$, the $CRII$ type is defined up to conjugation by the adjoint group of $\mathfrak{g}_0$ in terms of Cartan subalgebras and root systems. In fact, every subalgebra of type $CRII$ is of type $CRI$. 