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Classification des Structures CR Invariantes pour les Groupes de Lie Compacts

Let G_0 be a compact Lie group of dimension N whose Lie algebra is $\mathfrak{g}_0$. The notion of CR structure on a C^∞ manifold is known a long time ago. In this note we are interested by the CR stuctures on G_0 which are invariant by the left action of the group on the tangent bundle and which are of maximal rank. Such a structure is defined by its fibre $\mathfrak{h}$ at the neutral element which is a subalgebra of the complexification $\mathfrak{g}$ of $\mathfrak{g}_0$ whose dimension is the entire part $[N/2]$ of $N/2$ and whose intersection with $\mathfrak{g}_0$ is equal to $\{0\}$. Up to conjugation by the adjoint group of $\mathfrak{g}_0$, these subalgebras are classified. When N is even, there is only one type, type $CR0$. When N is odd, there are two types, type $CR0$ and type CRI. These types are given in terms of Cartan subalgebras and root systems. In any case, these subalgebras are solvable. Following M. S. Baouendi, L. P. Rothschild and F. Treves ["CR structures with group action and extendability of CR functions", Inventiones Mathematicae 82 (1985) 359–396], we introduce the notion of CR structures which are G_0-invariant and invariant by the transverse action of a G_0-invariant Lie subgroup. When this group is commutative, we get the notion of G_0-rigidity. We then prove, when N is odd, that a G_0-invariant CR structure, of maximal rank, is G_0-rigid if and only if the fibre of the CR structure at the neutral element, is of type $CR0$. Following H. Jacobowitz ["The canonical bundle and realizable CR hypersurfaces", Pacific Journal of Mathematics 127 (1987) 91–101], we introduce the canonical fibre bundle K of a G_0-invariant CR structure, of maximal rank, when N is odd. We prove that K contains a closed G_0-invariant form if and only if the fibre of the CR structure at the neutral element, is of type $CR0$ or type $CRII$. As for type $CR0$ and CRI, the $CRII$ type is defined up to conjugation by the adjoint group of $\mathfrak{g}_0$ in terms of Cartan subalgebras an root systems. In fact, every subalgebra of type $CRII$ is of type CRI.