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Remark on the Complexified Iwasawa Decomposition

Let $G_{\mathbb{R}}$ be a real form of a complex semisimple Lie group $G_{\mathbb{C}}$. We identify the complexification $K_{\mathbb{C}}A_{\mathbb{C}}N_{\mathbb{C}} \subset G_{\mathbb{C}}$ of an Iwasawa decomposition $G_{\mathbb{R}} = K_{\mathbb{R}}A_{\mathbb{R}}N_{\mathbb{R}}$ as $\{g \in G_{\mathbb{C}} \mid gB \in \Omega\}$ where $B \subset G_{\mathbb{C}}$ is a Borel subgroup of $G_{\mathbb{C}}$ that contains $A_{\mathbb{R}}N_{\mathbb{R}}$ and Ω is the open $K_{\mathbb{C}}$–orbit on $G_{\mathbb{C}}/B$. This is done in the context of subsets $K_{\mathbb{C}}R_{\mathbb{C}} \subset G_{\mathbb{C}}$, where $R_{\mathbb{C}}$ is a parabolic subgroup of $G_{\mathbb{C}}$ defined over $\mathbb{R}$, and the open $K_{\mathbb{C}}$–orbits on complex flag manifolds $G_{\mathbb{C}}/Q$.