Journal of Lie Theory 12 (2002) 265–288

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On the Structure of Graded Transitive Lie Algebras

We study finite-dimensional Lie algebras $\mathfrak{L}$ of polynomial vector fields in n variables that contain the vector fields $\dfrac{\partial}{\partial x_i}$ $(i = 1, \ldots, n)$ and $x_1\dfrac{\partial}{\partial x_1} + \cdots + x_n\dfrac{\partial}{\partial x_n}$.

We show that the maximal ones always contain a semi-simple subalgebra $\bar{\mathfrak{g}}$, such that $\dfrac{\partial}{\partial x_i} \in \bar{\mathfrak{g}}$ $(i = 1, \ldots, m)$ for an m with $1 \leq m \leq n$. Moreover a maximal algebra has no trivial $\bar{\mathfrak{g}}$-modules in the space spanned by $\dfrac{\partial}{\partial x_i} (i = m+1, \ldots, n)$.

The possible algebras $\bar{\mathfrak{g}}$ are described in detail, as well as all $\bar{\mathfrak{g}}$-modules that constitute such maximal $\mathfrak{L}$. The maximal algebras are described explicitly for $n \leq 3$.

Keywords: Lie algebras, vector fields, graded Lie algebras.

MSC: 17B66; 17B70, 17B05