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On the Combinatorics of Inflexion Arches of Saddle Spheres

Each saddle sphere $\Gamma \subset S^3$ is known to generate a spanning arrangement of at least four non-crossing oriented great semicircles on S^2. Each semicircle arises as the projection of an inflexion arch of the surface Γ. In the paper we prove the converse: each spanning arrangement of non-crossing oriented great semicircles is generated by some smooth saddle sphere. In particular, this means the diversity of saddle spheres on S^3. Recall that each C^2-smooth saddle sphere leads directly to a counterexample to the following conjecture of A. D. Alexandrov:
Let $K \subset \mathbb{R}^3$ be a smooth convex body. If, for a constant C, at every point of ∂K, we have $R_1 \leq C \leq R_2$, then K is a ball (R_1 and R_2 stand for the principal curvature radii of ∂K).
In the framework of the conjecture, the main result of the paper means that all counterexamples can be classified by non-crossing arrangements of oriented great semicircles.

Keywords: Alexandrov's conjecture, inflexion point, inflexion arch, saddle surface, hyperbolic virtual polytope.

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