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On Wallace Loci from the Projective Point of View

Let π_k be the projection of an n-dimensional projective space Σ $(2 \leq n < \infty)$ from the point B_k onto the hyperplane α_k, $k = 1, \ldots, n+1$, and assume that $\alpha_1, ..., \alpha_{n+1}$ are linearly independent. By the Wallace locus of $\pi_1, ..., \pi_{n+1}$ we mean the set of all points X of Σ whose images $\pi_1(X), ..., \pi_{n+1}(X)$ are linearly dependent. In a Pappian n-space each Wallace locus is either the entire space or an algebraic hypervariety whose degree is at most n+1. In a Pappian plane a triangle B_1, B_2, B_3 and a trilateral $\alpha_1, \alpha_2, \alpha_3$ determine the same Wallace locus as the triangle $\alpha_2 \cap \alpha_3, \alpha_3 \cap \alpha_1, \alpha_1 \cap \alpha_3$ and the trilateral $B_2 \vee B_3, B_3 \vee B_1, B_1 \vee B_2$. An analogous exchange rule for $3 \leq n < infty$ is not valid. For Wallace loci of a Pappian plane with collinear centers B_1, B_2, B_3 we exhibit a theorem wherefrom we get the Wallace theorems for all degenerate Cayley-Klein planes by specialization. Thus we get the orthogonal and oblique Euclidean Wallace lines, the orthogonal and oblique pseudo-Euclidean Wallace lines, and the isotropic Wallace lines and, by duality, the Wallace points of the dual-Euclidean plane, of the dual-pseudo-Euclidean plane, and of the isotropic plane.

Keywords: Triangle geometry, Wallace line, pedal line, Simson line, Wallace subspace.

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