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C. Jimenez Univ Brest, CNRS UMR 6205, Laboratoire de Mathématiques de Bretagne Atlantique, Brest, France chloe.jimenez@univ-brest.fr

Equivalence between Strict Viscosity Solution and Viscosity Solution in the Wasserstein Space and Regular Extension of the Hamiltonian in L^2_P

This article aims to build bridges between several notions of viscosity solution of first order dynamic Hamilton-Jacobi equations. The first main result states that, under assumptions, the definitions of Gangbo-Nguyen-Tudorascu and Marigonda-Quincampoix are equivalent. Secondly, to make the link with Lions' definition of solution, we build a regular extension of the Hamiltonian in $L_{\mathbb{P}}^2 \times L_{\mathbb{P}}^2$. This extension allows to give an existence result of viscosity solution in the sense of Gangbo-Nguyen-Tudorascu, as a corollary of the existence result in $L_{\mathbb{P}}^2 \times L_{\mathbb{P}}^2$. We also give a comparison principle for rearrangement invariant solutions of the extended equation. Finally we illustrate the interest of the extended equation by an example in Multi-Agent Control.

Keywords: Optimal transport, viscosity solutions, Hamilton-Jacobi equations, multi-agent optimal control.

MSC: 49L25.