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Finite Element Approximation of the Hardy Constant

We consider finite element approximations to the optimal constant for the Hardy inequality with exponent $p = 2$ in bounded domains of dimension $n = 1$ or $n \geq 3$. For finite element spaces of piecewise linear and continuous functions on a mesh of size h, we prove that the approximate Hardy constant converges to the optimal Hardy constant at a rate proportional to $1/|\log h|^2$. This result holds in dimension $n = 1$, in any dimension $n \geq 3$ if the domain is the unit ball and the finite element discretization exploits the rotational symmetry of the problem, and in dimension $n = 3$ for general finite element discretizations of the unit ball. In the first two cases, our estimates show excellent quantitative agreement with values of the discrete Hardy constant obtained computationally.

Keywords: Hardy inequality, Hardy constant, finite element method.

MSC: 46E35, 65N30.