A Varifold Perspective on the \( p \)-Elastic Energy of Planar Sets

Under suitable regularity assumptions, the \( p \)-elastic energy of a planar set \( E \subset \mathbb{R}^2 \) is defined as
\[
\mathcal{F}_p(E) = \int_{\partial E} 1 + |k_{\partial E}|^p \, d\mathcal{H}^1,
\]
where \( k_{\partial E} \) is the curvature of the boundary \( \partial E \). In this work we use a varifold approach to investigate this energy, that can be well defined on varifolds with curvature. First we show new tools for the study of 1-dimensional curvature varifolds, such as existence and uniform bounds on the density of varifolds with finite elastic energy. Then we characterize a new notion of \( L^1 \)-relaxation of this energy by extending the definition of regular sets by an intrinsic varifold perspective, also comparing this relaxation with the classical one of G. Bellettini and L. Mugnai [Characterization and representation of the lower semicontinuous envelope of the elastica functional, Annales de l’Institut Henri Poincaré (C), Non Linear Analysis 21(6) (2004) 839–880; A varifolds representation of the relaxed elastica functional, J. Convex Analysis 14(3) (2007) 543–564]. Finally we discuss an application to the inpainting problem, examples and qualitative properties of sets with finite relaxed energy.

Keywords: Curvature varifolds, \( p \)-elastic energy, relaxation.

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