

© 2019 Heldermann Verlag
Journal of Convex Analysis 26 (2019) 1337–1346

J. C. Ferrando

Centro de Investigación Operativa, Universidad Miguel Hernández, 03202 Elche, Spain
jc.ferrando@umh.es

J. Kąkol

Faculty of Mathematics and Informatics, A. Mickiewicz University, 61-614 Poznan, Poland
and: Institute of Mathematics, Czech Academy of Sciences, Prague, Czech Republic
kakol@amu.edu.pl

Metrizable Bounded Sets in $C(X)$ Spaces and Distinguished $C_p(X)$ Spaces

Quite recently W. Ruess [*Locally convex spaces not containing ℓ_1* , *Funct. Approx. Comment. Math.* 50 (2014) 351–358] has shown that a wide class of locally convex spaces for which all bounded sets are metrizable enjoy Rosenthal’s ℓ_1 -dichotomy. Being motivated by this fact we show that for a Tychonoff space X the bounded sets of $C_p(X)$ are metrizable (respectively, the bounded sets of $C_k(X)$ are weakly metrizable) if and only if X is countable. If X is a P -space we show that every bounded set in $C_p(X)$ is metrizable if and only if X is countable and discrete. The second part of the paper deals with distinguished $C_p(X)$ spaces. Among other things we show that $C_p(X)$ is distinguished if and only if the strong topology of the dual coincides with its strongest locally convex topology, and that $C_p(X)$ is always distinguished whenever X is countable.

Keywords: Countable tightness, Frechet-Urysohn space, strong dual, strongest locally convex topology, distinguished space.

MSC: 54C35, 54E15, 46A03