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A Necessary and Sufficient Condition on the Stability of the Infimum of Convex Functions

Let us say that a convex function $f: C \to [-\infty, \infty]$ on a convex set $C \subseteq \mathbb{R}$ is infimum-stable if, for any sequence (f_n) of convex functions $f_n: C \to [-\infty, \infty]$ converging to f pointwise, one has

$$\inf_C f_n \to \inf_C f.$$

A simple necessary and sufficient condition for a convex function to be infimumstable is given. The same condition remains necessary and sufficient if one uses Moore-Smith nets (f_{ν}) in place of sequences (f_n) . This note is motivated by certain applications to stability of measures of risk/inequality in finance/economics.

Keywords: Convex functions, minimization, stability, convergence, Legendre-Fenchel transform.

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