(Quasi)additivity Properties of the Legendre-Fenchel Transform and its Inverse, with Applications in Probability

The notion of the Hölder convolution is introduced. The main result is that, under general conditions on functions $L_1, \ldots, L_n$, one has

$$(L_1 \star \cdots \star L_n)^{\ast -1} = L_1^{\ast -1} + \cdots + L_n^{\ast -1},$$

where $\star$ denotes the Hölder convolution and $L^{\ast -1}$ is the function inverse to the Legendre-Fenchel transform $L^\ast$ of a given function $L$. General properties of the functions $L^\ast$ and $L^{\ast -1}$ are discussed. Applications to probability theory are presented. In particular, an upper bound on the quantiles of the distribution of the sum of (possibly dependent) random variables is given.

**Keywords:** Hölder convolution, Legendre-Fenchel transform, probability inequalities, exponential inequalities, sums of random variables, exponential rate function, Cramer-Chernoff function, quantiles.

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