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(Quasi)additivity Properties of the Legendre-Fenchel Transform and its Inverse, with Applications in Probability

The notion of the Hölder convolution is introduced. The main result is that, under general conditions on functions $L_1, \dots, L_n$, one has

$$(L_1 \boxed{\scriptstyle H} \cdots \boxed{\scriptstyle H} L_n)^{*-1} = {L_1}^{*-1} + \cdots + {L_n}^{*-1},$$

where $\boxed{\scriptstyle H}$ denotes the Hölder convolution and L^{*-1} is the function inverse to the Legendre-Fenchel transform L^* of a given function L. General properties of the functions L^* and L^{*-1} are discussed. Applications to probability theory are presented. In particular, an upper bound on the quantiles of the distribution of the sum of (possibly dependent) random variables is given.

Keywords: Hoelder convolution, Legendre-Fenchel transform, probability inequalities, exponential inequalities, sums of random variables, exponential rate function, Cramer-Chernoff function, quantiles.

MSC: 26A48, 26A51; 60E15