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The Monotonicity of Perturbed Gradients of Convex Functions

We perturb the (Minty-Browder monotone) gradients of some convex functions f, defined on convex open subsets of the Euclidean space with suitable linear transformations A and provide sufficient conditions on f and A in order to obtain the η-monotonicity of the perturbation $\nabla f + A$ for some $\eta \in (-1, 0)$. These conditions are given in terms of the magnitude of the Hessian operators associated to f and the spectra of the operators $(A + A^*)/2$ and A^*A and they make the Minty-Browder monotonicity of the perturbation $\nabla f + A$ to fail. While the symmetric perturbations are only considered as instructive examples in the Minty-Browder monotone case, we mostly rely, in the even dimensional context, on linear transformations which can be represented as linear combinations between the identity operator and skew-symmetric isometries. Note that the skew-symmetric isometries of $\mathbb{R}^{2m}$ are complex structures on $\mathbb{R}^{2m}$. In the bidimensional case the rotations do admit such representations and the perturbations of the gradients of convex functions by rotations were tretead before by D. Marian, I. R. Peter, and C. Pintea, *A class of generalized monotone operators*, J. Math. Anal. Appl. 421 (2015) 1827–1843. We also obtain some global injectivity results for suitable linear perturbations of some gradients of convex functions.

Keywords: Minty-Browder monotone operator, eta-monotone operator, h-monotone operator.

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