Journal of Convex Analysis 24 (2017) 107–122

**E. Parini**
Centre de Mathématiques et Informatique, Aix-Marseille University, 39 rue F. Joliot Curie, 13453 Marseille Cedex 13, France
enea.parini@univ-amu.fr

# Reverse Cheeger Inequality for Planar Convex Sets

We prove the sharp inequality

$$J(\Omega) := \frac{\lambda_1(\Omega)}{h_1(\Omega)^2} < \frac{\pi^2}{4},$$

where $\Omega$ is any planar, convex set, $\lambda_1(\Omega)$ is the first eigenvalue of the Laplacian under Dirichlet boundary conditions, and $h_1(\Omega)$ is the Cheeger constant of $\Omega$. The value on the right-hand side is optimal, and any sequence of convex sets with fixed volume and diameter tending to infinity is a maximizing sequence. Morever, we discuss the minimization of $J$ in the same class of subsets: we provide a lower bound which improves the generic bound given by Cheeger's inequality, we show the existence of a minimizer, and we give some optimality conditions.

**Keywords**: Cheeger's inequality.

**MSC**: 49Q10