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J. Kwon

Institut de Mathématiques, équipe Combinatoire et Optimisation, Université Pierre-et-Marie-Curie, 4 place Jussieu, 75252 Paris Cedex 05, France
joon.kwon@ens-lyon.org

A Universal Bound on the Variations of Bounded Convex Functions

Given a convex set C in a real vector space E and two points $x, y \in C$, we investigate which are the possible values for the variation $f(y) - f(x)$, where $f : C \rightarrow [m, M]$ is a bounded convex function. We then rewrite the bounds in terms of the Funk weak metric, which will imply that a bounded convex function is Lipschitz-continuous with respect to the Thompson and Hilbert metrics. The bounds are also proved to be optimal. We also exhibit the maximal sub-differential of a bounded convex function at a given point $x \in C$.

Keywords: Convex functions, variations, Funk metric, Thompson metric, Hilbert metric.

MSC: 26B25, 52A05