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Splitting Forward-Backward Penalty Scheme for Constrained Variational Problems

We study a forward backward splitting algorithm that solves the variational inequality

$$Ax + \nabla\Phi(x) + N_C(x) \ni 0$$

where $\mathcal{H}$ is a real Hilbert space, $A : \mathcal{H} \rightrightarrows \mathcal{H}$ is a maximal monotone operator, $\Phi : \mathcal{H} \to \mathbf{R}$ is a smooth convex function, and N_C is the outward normal cone to a closed convex set $C \subset \mathcal{H}$. The constraint set C is represented as the intersection of the sets of minima of two convex penalization function $\Psi_1 : \mathcal{H} \to \mathbf{R}$ and $\Psi_2 : \mathcal{H} \to \mathbf{R} \cup \{+\infty\}$. The function Ψ_1 is smooth, the function Ψ_2 is proper and lower semicontinuous. Given a sequence (β_n) of penalization parameters which tends to infinity, and a sequence of positive time steps (λ_n), the algorithm (SFBP), $n \geq 1$,

$$\left\{\begin{array}{rcl} x_1 & \in & \mathcal{H}, \\ x_{n+1} & = & (I + \lambda_n A + \lambda_n\beta_n\partial\Psi_2)^{-1}(x_n - \lambda_n\nabla\Phi(x_n) - \lambda_n\beta_n\nabla\Psi_1(x_n)), \end{array}\right.$$

performs forward steps on the smooth parts and backward steps on the other parts. Under suitable assumptions, we obtain weak ergodic convergence of the sequence (x_n) to a solution of the variational inequality. Convergence is strong when either A is strongly monotone or Φ is strongly convex. We also obtain weak convergence of the whole sequence (x_n) when A is the subdifferential of a proper lower semicontinuous convex function. This provides a unified setting for several classical and more recent results, in the line of historical research on continuous and discrete gradient-like systems.

Keywords: Constrained convex optimization, forward-backward algorithms, hierarchical optimization, maximal monotone operators, penalization methods, variational inequalities.

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