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Strictly Convex Space: Strong Orthogonality and Conjugate Diameters

In a normed linear space X an element x is said to be orthogonal to another element y in the sense of Birkhoff-James, written as $x \perp_B y$, iff $\|x\| \leq \|x + \lambda y\|$ for all scalars λ. We prove that a normed linear space X is strictly convex iff for any two elements x, y of the unit sphere S_X, $x \perp_B y$ implies $\|x + \lambda y\| > 1$ for all $\lambda \neq 0$. We apply this result to find a necessary and sufficient condition for a Hamel basis to be strongly orthonormal in the sense of Birkhoff-James in a finite dimensional real strictly convex space X. Applying the result we give estimations for the lower bounds of $\|tx + (1-t)y\|$, $t \in [0,1]$ and $\|y + \lambda x\|$, for all λ and for all elements $x, y \in S_X$ with $x \perp_B y$. We find a necessary and sufficient condition for the existence of conjugate diameters through the points $e_1, e_2 \in S_X$ in a real strictly convex space of dimension 2. The concept of generalized conjugate diameters is then developed for a real strictly convex smooth space of finite dimension.

Keywords: Orthogonality, strict convexity, extreme point, conjugate diameters.

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