D. Azagra
ICMAT, Dep. de Análisis Matemático, Facultad Ciencias Matemáticas, Universidad Complutense, 28040 Madrid, Spain
`azagra@mat.ucm.es`

C. Mudarra
ICMAT, Calle Nicolás Cabrera 13-15, Campus de Cantoblanco, 28049 Madrid, Spain
`carlos.mudarra@icmat.es`

Global Approximation of Convex Functions by Differentiable Convex Functions on Banach Spaces

We show that if X is a Banach space whose dual X^* has an equivalent locally uniformly rotund (LUR) norm, then for every open convex $U \subseteq X$, for every real number $\varepsilon > 0$, and for every continuous and convex function $f : U \to \mathbb{R}$ (not necessarily bounded on bounded sets) there exists a convex function $g : U \to \mathbb{R}$ of class $C^1(U)$ such that $f - \varepsilon \le g \le f$ on U. We also show how the problem of global approximation of *continuous* (not necessarily bounded on bounded sets) convex functions by C^k smooth convex functions can be reduced to the problem of global approximation of *Lipschitz* convex functions by C^k smooth convex functions.

Keywords: Approximation, convex function, differentiable function, Banach space.

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