

© 2015 Heldermann Verlag
Journal of Convex Analysis 22 (2015) 493–520

M. Bounkhel

Department of Mathematics, King Saud University, P. O. Box 2455, Riyadh 11451, Saudi-Arabia
bounkhel@ksu.edu.sa

On Subdifferentials of a Minimal Time Function in Hausdorff Topological Vector Spaces at Points Outside the Target Set

The present paper is a continuation of the study of the minimal time functions in Hausdorff topological vector spaces, started by the author in a previous paper [On subdifferentials of a minimal time function in topological vector spaces, *Applicable Analysis: An Internat. Journal*, DOI: 10.1080/00036811.2013.848271]. We consider in this work the case of points outside the target set and we prove and extend various important properties on directional derivatives and subdifferentials of $T_{S,\Omega}$ at points $x \notin S$ in the convex and nonconvex cases. These results are used to prove various new characterizations of the convex tangent cone, Clarke tangent cone, Bouligand tangent cone, and Clarke normal cone to the enlargement $S(r)$ ($r := T_{S,\Omega}(\bar{x}) > 0$) of S at $\bar{x} \notin S$ in terms of the minimal time function at \bar{x} , in Hausdorff topological vector spaces. Our results extend various existing results, in convex and nonconvex cases, from Banach spaces and normed vector spaces to Hausdorff topological vector spaces. Even in Banach spaces and in normed vector spaces our results are new and they extend various existing results on the distance function to closed sets by taking Ω to be the closed unit ball. Applications to normal and subdifferential regularities in normed vector spaces are also given in the last section of the paper.

Keywords: Minimal time function, Clarke tangent cone, Clarke normal cone, tangential regularity, directional regularity.

MSC: 49J52, 90C26