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Extension of Continuous Convex Functions from Subspaces I

Let X be a topological vector space, $Y \subset X$ a subspace, and $A \subset X$ an open convex set containing 0. We are interested in the extendability of a continuous convex function $f\colon A \cap Y \to \mathbb{R}$ to a continuous convex function $F\colon A \to \mathbb{R}$. We characterize such extendability: (a) for a given f; (b) for every f. The case (b) for $A = X$ generalizes results from a paper by J. Borwein, V. Montesinos and J. Vanderwerff [Boundedness, differentiability and extensions of convex functions, J. Convex Analysis 13 (2006) 587–602], and from another one by L. Zajíček and the second author [On extensions of d.c. functions and convex functions, J. Convex Analysis 17 (2010) 427–440]. We also show that if X is locally convex and X/Y is "conditionally separable", then the couple (X, Y) satisfies the CE-property, saying that the above extendability holds for $A = X$ and every f. It follows that every couple (X, Y) has the CE-property for the weak topology.
We consider also a stronger SCE-property saying that the above extendability is true for every A and every f. A deeper study of the SCE-property will appear in a subsequent paper.

Keywords: Convex function, extension, topological vector space, normed linear space.

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