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Measures of Weak Noncompactness in Non-Archimedean Banach Spaces

Let E be a non-Archimedean Banach space over a non-Archimedean locally compact non-trivially valued field $\mathbb{K} := (\mathbb{K}, |.|)$. Let E'' be its bidual and M a bounded set in E. We say that M is ε-weakly relatively compact if $\overline{M}^{\sigma(E'',E')} \subset E + B_{E'',\varepsilon}$, where $B_{E'',\varepsilon}$ is the closed ball in E'' with the radius $\varepsilon \geq 0$. In this paper we describe measures of noncompactness γ, k and De Blasi measure ω. We show that $\gamma(M) \leq k(M) \leq \omega(M) = \omega(acoM) \leq \frac{1}{|\rho|}\gamma(M)$, where ρ ($|\rho| < 1$) is an uniformizing element in $\mathbb{K}$, and $\omega(M) = \sup\{\overline{\lim}_m \ dist\,(x_m, [x_1, \ldots, x_{m-1}]) : (x_m) \subset M\}$; the latter equality is purely non-Archimedean. In particular, assuming $|\mathbb{K}| = \{||x|| : x \in E\}$, we prove that the absolutely convex hull $acoM$ of a $\varepsilon-$weakly relatively compact subset M in E is $\varepsilon-$weakly relatively compact. In fact we show that in this case for a bounded set M in E we have $\gamma(M) = \gamma(acoM) = k(M) = k(acoM) = \omega(M)$, Note that the above equalities fail in general for real Banach spaces by results of A. S. Granero [*An extension of the Krein-Smulian theorem*, Rev. Mat. Iberoam. 22 (2006) 93–100] and K. Astala and H. O. Tylli [*Seminorms related to weak compactness and to Tauberian operators*, Math. Proc. Cambridge Philos. Soc. 107 (1990) 367–375]. Most proofs are strictly non-Archimedean. A non-Archimedean variant of another quantitative Krein's theorem due to Fabian, Hajek, Montesinos and Zizler is also provided, see Corollary 9.

Keywords: Krein's theorem, Compactness, Measures of weak noncompactness.

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