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Mean-Value Inequalities for Convex Functions and the Chebysev-Vietoris Inequality

It is shown that if $B = [-b_1, b_1] \times \cdots \times [-b_n, b_n] \subset \mathbb{R}^n$, where $b_i > 0$ for $i = 1, ..., n$, and if A is a convex and compact subset of B of positive Lebesgue measure, which is preserved by reflections with respect to all coordinate hyperplanes $x_i = 0$ for $i = 1, ..., n$, then A is convexly majorized by B, i.e., for every continuous convex function v defined over B, the mean of v over A is not exceeding the mean of v over B. In the proof an n-dimensional extension of the integral form of the Chebysev inequality, which was given by L. Vietoris [*Eine Verallgemeinerung eines Satzes von Tschebyscheff*, Univ. Beograd Publ. Elektrotehn, Fak. Ser. Mat. Fiz 461-497 (1974) 115-117], is used.

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