Journal of Convex Analysis 21 (2014) 401–413

E. Kasior
Institute of Mathematics, University of Szczecin, Wielkopolska 15, 70–451 Szczecin 3, Poland
ekasior@univ.szczecin.pl

M. Wisła
Faculty of Mathematics and Computer Science, Adam Mickiewicz University, ul. Umultowska 87, 61-614 Poznan, Poland
marek.wisla@amu.edu.pl

Closedness of the Set of Extreme Points in Calderon-Lozanovskii Spaces

It is known [see R. M. Blumenthal, J. Lindenstrauss, R. R. Phelps, *Extreme operators into C(K)*, Pacific Journal of Mathematics 15(3) (1965), 747-756] that a compact linear operator from a Banach space X into the space of continuous functions $C(Z,\mathbb{R})$ is extreme provided it is nice, i.e. $T^*(Z) \subset \operatorname{Ext} B(X^*)$, where Z is a compact Hausdorff space and $T^* : Z \to X^*$ is a continuous function defined by $T^*(z)(x) = T(x)(z)$. The nice operator condition can be weakened as long as the set of extreme points $\operatorname{Ext} B(X^*)$ is closed, namely it suffices to assume than $T^*(Z_0) \subset \operatorname{Ext} B(X^*)$ for some dense subset $Z_0 \subset Z$ in that case. The aim of this paper is to characterize the closedness of the set of extreme points of the unit ball of Calderon-Lozanovskii spaces E_φ generated by the Köthe space E and the Orlicz function φ. The main theorem of the paper (Theorem 2.12) gives conditions under which the closedness of the set $\operatorname{Ext} B(E_\varphi)$ is equivalent to the closedness of the set of extreme points of the unit ball of the corresponding Köthe space E.

Keywords: Calderon-Lozanovskii spaces, extreme points, compact operators, Orlicz spaces, Koethe spaces.

MSC: 46B20, 46E30