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L. Cheng
School of Mathematical Sciences, Xiamen University, Xiamen 361005, P. R. China
lxcheng@xmu.edu.cn

Y. Zhou
School of Mathematical Sciences, Xiamen University, Xiamen 361005, P. R. China
roczhoufly@126.com

On Approximation by Δ-Convex Polyhedron Support Functions and the Dual of $cc(X)$ and $wcc(X)$

The classical Weierstrass theorem states that every continuous function f defined on a compact set $\Omega \subset \mathbb{R}^n$ can be uniformly approximated by polynomials. We show first that it is again valid if Ω is a compact Hausdorff metric space, i.e., it holds in the following sense: there exists a surjective isometry T from a compact set K_Ω of a Banach sequence space S to Ω, such that for every $\varepsilon > 0$ there is an n variable polynomial p satisfying

$$|f(T(s)) - p(s_1, s_2, \cdots, s_n)| < \varepsilon, \ \forall s = (s_j) \in K_\Omega.$$

We prove also that for any *weak* (w^*, resp.) continuous positively homogenous function f defined on a (dual, resp.) Banach space X (X^*, resp.) then for all $\varepsilon > 0$ and for every weakly compact set $K \subset X$(w^* compact set $K \subset X^*$), there exist $\phi_i \in X^*$ (X, resp.) for $i = 1, 2, \cdots, m$, and $\psi_j \in X^*$ (X, resp.) for $j = 1, 2, \cdots, n$ such that

$$|f(x) - [(\phi_1 \vee \phi_2 \vee \cdots \vee \phi_m)(x) - (\psi_1 \vee \psi_2 \vee \cdots \vee \psi_n)(x)]| < \varepsilon$$

uniformly for $x \in K$. Let $cc(X)$ ($wcc(X)$, reps.) be the norm semigroup consisting of all nonempty (weakly, resp.) compact convex sets of the space X. As its application, we give two representation theorems of the duals of $cc(X)$ and $wcc(X)$.

Keywords: Weierstrass theorem, function approximation, weakly continuous function, weakly compact set, normed semigroup, Delta-convex polyhedron support function.

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