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**Symmetry in Multi-Phase Overdetermined Problems**

We prove symmetry for a multi-phase overdetermined problem, with nonlinear governing equations. The most simple form of our problem (in the two-phase case) is as follows: For a bounded  $C^1$  domain  $\Omega \subset \mathbb{R}^n$  ( $n \geq 2$ ) let  $u^+$  be the Green's function (for the  $p$ -Laplace operator) with pole at some interior point (origin, say), and  $u^-$  the Green's function in the exterior with pole at infinity. If for some strictly increasing function  $F(t)$  (with some growth assumption) the condition  $\partial_\nu u^+ = F(\partial_\nu u^-)$  holds on the boundary  $\partial\Omega$ , then  $\Omega$  is necessarily a ball. We prove the more general multi-phase analog of this problem.

**Keywords:** Symmetry, overdetermined problems, multi-phases, viscosity solutions, Green's function.

**MSC:** 35R35, 35B06