V. Soltan

Dept. of Mathematical Sciences, George Mason University, Fairfax, VA 22030, U.S.A.
`vsoltan@gmu.edu`

A Characteristic Intersection Property of Generalized Simplices

Following R. T. Rockafellar ["Convex Analysis", Princeton University Press, Princeton (1970)], a generalized n-simplex in $\mathbb{R}^n$ is defined as the direct sum of an m-simplex and a simplicial $(n-m)$-cone, $0 \leq m \leq n$. R. Fourneau ["Non-closed simplices and quasi-simplices", Mathematika 24 (1977) 71–85] showed that a line-free n-dimensional closed convex set $K \subset \mathbb{R}^n$ is a generalized n-simplex if and only if all n-dimensional intersections $K \cap (v + K)$, $v \in \mathbb{R}^n$, are homothetic to K. We extend this characteristic property by proving that for a pair of line-free n-dimensional closed convex sets K_1 and K_2 in $\mathbb{R}^n$ the following two conditions are equivalent: (1) all n-dimensional intersections $K_1 \cap (v + K_2)$, $v \in \mathbb{R}^n$, belong to a unique homothety class of convex sets, (2) K_1 and K_2 are generalized n-simplices whose n-dimensional intersections $K_1 \cap (v + K_2)$, $v \in \mathbb{R}^n$, are homothetic to a unique generalized n-simplex.

Keywords: Homothety, convex body, intersection, generalized simplex.

MSC: 52A20