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Journal of Convex Analysis 17 (2010) 925–943

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The Monge-Kantorovich Problem for Distributions and Applications

We study the Kantorovich-Rubinstein transhipment problem when the difference between the source and the target is not anymore a balanced measure but belongs to a suitable subspace $\mathbf{X}(\Omega)$ of first order distribution. A particular subclass $\mathbf{X}_0^\sharp(\Omega)$ of such distributions will be considered which includes the infinite sums of dipoles $\sum_k(\delta_{p_k}-\delta_{n_k})$ studied recently by A. C. Ponce ["On the distributions of the form $\sum_i(\delta_{p_i}-\delta_{n_i})$", C. R. Math. Acad. Sci. Paris 336 (2003) 571–576; and "On the distributions of the form $\sum_i(\delta_{p_i}-\delta_{n_i})$", J. Funct. Anal. 210 (2004) 391–435]. In spite of this weakened regularity, it is shown that an optimal transport density still exists among nonnegative finite measures. Some geometric properties of the Banach spaces $\mathbf{X}(\Omega)$ and $\mathbf{X}_0^\sharp(\Omega)$ can be then deduced.

Keywords: Monge-Kantorovich problem, optimal transportation, transhipment problem, flat norm, minimal connections, Jacobians.

MSC: 49J45, 49J20, 82C70, 90B06