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On Malamud Majorization and the Extreme Points of its Level Sets

We consider two types of majorization relationships between sequences of vectors $y = (y_k)_{k=1}^m$ and $x = (x_k)_{k=1}^\ell$ in $\mathbb{R}^n$ with $\ell \le m$. It is said that x is majorized by y, $x \prec y$, if the sum of any k vectors from x is in the convex hull of all possible sums of k vectors from y. It is said that x is doubly stochastically majorized by y, $x \prec_{\rm ds} y$, if $x_k = \sum_{j=1}^m m_{kj} y_j$, $k = 1, ..., \ell$, for some doubly stochastic matrix $M = (m_{kj})_{k,j=1}^{m,m}$.
In a recent article ["Inverse spectral problem for normal matrices and the Gauss-Lucas Theorem", Trans. Amer. Math. Soc. 357(10) (2004) 4043–4064] S. M. Malamud formulated the problem of finding a geometric condition guaranteeing that $x \prec y \Leftrightarrow x \prec_{\rm ds} y$. We answer this question in the case when the vectors in y are distinct and are extreme points of their convex hull. In particular, we derive a geometric characterization of the extreme points of the level set $L^2_\prec(y) = \{x : x \prec y\}$. Finally, we derive a set of algebraic conditions that characterize the extreme points of $L^\ell_\prec(y) = \{x : x \prec y\}$ for any $\ell \le m$ and y.

Keywords: Convex set, doubly stochastic majorization, Malamud majorization, extreme point, convex function, CVS class.

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