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Estimates on the Derivative of a Polynomial with a Curved Majorant Using Convex Techniques

A mapping $\phi\colon [-1,1] \to [0,\infty)$ is a curved majorant for a polynomial p in one real variable if $|p(x)| \leq \phi(x)$ for all $x \in [-1,1]$. If $\mathcal{P}_n^\phi(\mathbb{R})$ is the set of all one real variable polynomials of degree at most n having the curved majorant ϕ, then we study the problem of determining, explicitly, the best possible constant $\mathcal{M}_n^\phi(x)$ in the inequality

$$|p'(x)| \leq \mathcal{M}_n^\phi(x)\|p\|,$$

for each fixed $x \in [-1,1]$, where $p \in \mathcal{P}_n^\phi(\mathbb{R})$ and $\|p\|$ is the sup norm of p over the interval $[-1,1]$. These types of estimates are known as Bernstein type inequalities for polynomials with a curved majorant. The cases treated in this manuscript, namely $\phi(x) = \sqrt{1-x^2}$ or $\phi(x) = |x|$ for all $x \in [-1,1]$ (circular and linear majorant respectively), were first studied by Q. I. Rahman ["On a problem of Turán about polynomials with curved majorants", Trans. Amer. Math. Soc. 163 (1972) 447–455]. In that reference the author provided, for each $n \in \mathbb{N}$, the maximum of $\mathcal{M}_n^\phi(x)$ over $[-1,1]$ as well as an upper bound for $\mathcal{M}_n^\phi(x)$ for each $x \in [-1,1]$, where ϕ is either a circular or a linear majorant. Here we provide sharp Bernstein inequalities for some specific families of polynomials having a linear or circular majorant by means of classical convex analysis techniques (in particular we use the Krein-Milman approach).