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Quasiconvexity and Uniqueness of Stationary Points on a Space of Measure Preserving Maps

Let $\Omega \subset \mathbb{R}^n$ be a bounded starshaped domain and consider the energy functional

$$\mathbb{F}[u;\Omega] := \int_\Omega \mathbf{F}(\nabla u(x))\,dx,$$

over the space of measure preserving maps

$$\mathcal{A}_p(\Omega) = \left\{ u \in \bar{\xi}x + W_0^{1,p}(\Omega,\mathbb{R}^n) : \det \nabla u = 1 \ a.e. \text{ in } \Omega \right\},$$

with $p \in [1,\infty[$, $\bar{\xi} \in \mathbb{M}_{n\times n}$ and $\det \bar{\xi} = 1$. In this short note we address the question of *uniqueness* for solutions of the corresponding system of Euler-Lagrange equations. In particular we give a new proof of the celebrated result of R. J. Knops and C. A. Stuart [Arch. Rational Mech. Anal. 86, No. 3 (1984) 233–249] using a method based on *comparison* with homogeneous degree-one extensions as introduced by the second author in his recent paper "Quasiconvexity and uniqueness of stationary points in the multi-dimensional calculus of variations" [Proc. Amer. Math. Soc. 131, (2003) 3101–3107].