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**Deville’s Master Lemma and Stone’s Discreteness in Renorming Theory**

Banach spaces  $X$  with an equivalent  $\sigma(X, F)$ -lower semicontinuous and locally uniformly rotund norm, for a norming subspace  $F \subset X^*$ , are those spaces  $X$  that admit countably many families of convex and  $\sigma(X, F)$ -lower semicontinuous functions  $\{\varphi_i^n : X \rightarrow \mathbb{R}^+; i \in I_n\}_{n=1}^\infty$  such that there are open subsets

$$G_i^n \subset \{\varphi_i^n > 0\} \cap \{\varphi_j^n = 0 : j \neq i, j \in I_n\}$$

with  $\{G_i^n : i \in I_n, n \in \mathbb{N}\}$  a basis for the norm topology of  $X$ .

**Keywords:** Banach space, local uniform rotundity, slicely-isolatedness, network, convex biorthogonal system.

**MSC:** 46B03, 46B20, 46B26, 54E35