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Maximal Monotone Operators with a Unique Extension to the Bidual

We study a sufficient condition under which a maximal monotone operator $T\colon X \rightrightarrows X^*$ admits a unique maximal monotone extension to the bidual $\widetilde{T}\colon X^{**} \rightrightarrows X^*$. We will prove that for non-linear operators this condition is equivalent to uniqueness of the extension. The central tool in our approach is the $\mathcal{S}$-function defined and studied previously by R. S. Burachik and B. F. Svaiter ["Maximal monotone operators, convex functions and a special family of enlargements", Set-Valued Analysis 10 (2002) 297–316]. For a generic operator, this function is the supremum of all convex lower semicontinuous functions which are majorized by the duality product in the graph of the operator.
We also prove in this work that if the graph of a maximal monotone operator is convex, then this graph is an affine linear subspace.

Keywords: Maximal monotone operators, extension, bidual, Banach spaces, Broendsted-Rockafellar property, S-function.

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