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Heat Flow for Closed Geodesics on Finsler Manifolds

We use the celebrated heat flow method of Eells and Sampson to the question of deformation of a smooth loop $M \in \mathbf{R}^2$ on a Finsler manifold (N, h) to a closed geodesic in N. This leads to the investigation of the corresponding heat equation which is the parabolic initial value problem

$$\begin{aligned} \frac{\partial u^{i}}{\partial t} &- \frac{\partial^{2} u^{i}}{\partial x^{2}} &= \Gamma^{i}_{hk} \left(u, \frac{\partial u}{\partial x} \right) \frac{\partial u^{h}}{\partial x} \frac{\partial u^{k}}{\partial x} \text{ in } M \times [0, T), \\ u \left(x, 0 \right) &= f \left(x \right); \; i = 1, ..., n. \end{aligned}$$

The existence of a global in time solution u(x,t) and its subsequent convergence to a closed geodesic $u_{\infty}: M \to N$ as $t \to \infty$, are dealt with. Appropriate concepts arising from the Finslerian nature of the problem are introduced.