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Thomas Lachand-Robert

Memorial Volume

Guillaume Carlier
CEREMADE, Université Paris Dauphine
carlier@ceremade.dauphine.fr

Myriam Comte
Laboratoire Jacques-Louis Lions, Université Pierre et Marie Curie
comte@ann.jussieu.fr

On February 23rd, 2006, our colleague, collaborator and dear friend, Thomas Lachand-Robert died in a tragic domestic accident. Thomas was only 39. While writing these lines, we have a very special thought for his wife, Martine. Since Thomas' death, which was a terrible shock for all of us, Thomas' friends received an impressive number of messages of support and sympathy and testimonies from all other the world, which shows how respected and influential Thomas was.

1. Introduction

Thomas was born in Paris on December 18th 1966. Thomas was relatively discreet regarding his childhood but sometimes alluded to his parents' divorce which actually affected him a lot. At high school, Thomas was very interested and talented in Mathematics and Physics but also discovered a passion for History. After his *baccalauréat* obtained with honours in 1983, Thomas entered the lycée Louis-Le Grand in Paris to prepare the *concours d'entrée aux grandes écoles scientifiques*. In 1986, he was admitted at the most prestigious french Grandes Écoles, namely the École Normale Supérieure de Paris and the École Polytechnique and Thomas chose the latter. At that period, Thomas developed his taste for computer science and discovered the possibilities offered to mathematicians by scientific programming. Thomas published several books on scientific, object oriented, C++ and Turbo Pascal programming between 1985 and 1991. As recently recalled by André Warusfel, in a first version of one of these books (written by Thomas while he was at Louis-Le Grand, that is, at a time where usually one does not have much time left), Thomas presented himself as "preparing the École Polytechnique", which gives an idea of Thomas' strong character (and self confidence). In 1990, Thomas obtained with the highest honours the DEA d'analyse numérique from the university Pierre and Marie Curie, Paris 6. He then started a Ph.D thesis under the supervision of Henri Berestycki on qualitative properties of solutions of semi-linear elliptic partial differential equations. He defended his thesis entitled "Équations aux dérivées partielles elliptiques semi-linéaires; propriétés de monotonie, réarrangement et ruptures de symétrie" and obtained his Ph.D with the highest honours in 1993. Thomas then became maître de conférences (assistant professor) at the Laboratoire d'Analyse Numérique de l'Université de Paris 6 (now Laboratoire Jacques-Louis Lions) where he stayed until 2001. In 2000, Thomas defended his *Habilitation à diriger des recherches*, gathering a series of papers devoted to unusual

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variational problems subject to global constraints, motivated by Newton's problem of minimal resistance. In 2001, Thomas obtained a full professor position at the Université de Savoie in Chambéry where he became the director of the LAMA (Laboratoire de Mathématiques) in 2005.

Throughout his career, Thomas was recognized for his energy and enthusiasm, not only in research but also in his teaching and administrative activities. His colleagues, in Paris and Chambéry remember well his leading and passionate character, his vast culture and his numerous centers of interest. Thomas wrote several reference books on TeX and LaTeX, and, on that subject as well on so many others, we learnt so much from him. His qualities as director of the LAMA were in particular noticed by the evaluating committee of the CNRS a few days before Thomas left us. Thomas organized or co-organized several successful meetings. We're pretty sure that all participants of the "Calculus of Variations" meeting in Chambéry in June 2003 remember the heat wave, the cruise on the Bourget lake and the fine restaurants Thomas had carefully chosen.

As a mathematician, Thomas had a particular taste for difficult and original problems where he could use his skills in clever geometric constructions. On several problems he was interested in, Thomas developed original numerical methods, which often lead him to formulate interesting conjectures. From the late 1990's, Thomas got mainly interested in shape optimization and calculus of variations with global constraints. He quickly became an expert in the field and drove many collaborators on the subject. Our aim is to describe now some aspects of Thomas' scientific interests. We refer to the bibliography for a complete list of references.

Newton's body of least resistance problem may be formulated as

$$\inf R(u) := \int_\Omega \frac{1}{1+|\nabla u(x)|^2}\,\mathrm{d}x \;\text{ subject to: } u \text{ concave } 0 \le u \le M$$

where Ω is a bounded convex subset of $\mathbb{R}^d$, the unknown function u models the body and $M \geq 0$ gives a height constraint. The concavity constraint ensures that incident particles hit the body only once. Thomas interest in variational problems subject to global constraints (such as concavity) probably originated with the discovery by Brock, Kawohl and Ferone that, when Ω is the ball of $\mathbb{R}^2$, the solution of Newton's minimal resistance problem was not radial. The solution computed by Newton himself is then optimal in the class of rotationally invariant convex bodies but not optimal among all convex bodies, and the determination of such optimal bodies is still an open question today. For Thomas who was working on symmetry results in elliptic PDE's, this symmetry breaking phenomenon was very intriguing and he found in this type of problems a challenging area of research. He first worked on the subject with Mark Peletier and later with various coauthors: Guillaume Carlier, Myriam Comte, Bertrand Maury, and Édouard Oudet.

In 2001, in collaboration with Ioan Ionescu and his team, Thomas started working on landslides modelling based on an inhomogeneous Bingham fluid model. Surprisingly, Thomas related a blocking property of the fluid to Cheeger's problem and Cheeger's constant. Given a regular bounded domain Ω the problem consists in finding a set of finite perimeter $E \subset \overline{\Omega}$ minimizing the ratio perimeter over volume. The value of this problem is called the Cheeger constant of Ω and minimizers are called Cheeger sets of Ω.

In landslides modelling, due to inhomogeneities, it is relevant to generalize this problem to the case of weighted perimeter and weighted volume. This Cheeger problem has many interesting applications and connections to the continuous max-flow/min cut theorem, to image processing... Faber-Krahn inequality gives a lower bound for the first eigenvalue of the p-laplacian of Ω in terms of its Cheeger constant (first eigenvalue of the 1-laplacian). Not only did Thomas realize the connection between Cheeger sets and landslides, but he also gave, in collaboration with Bernd Kawohl, an explicit construction for Cheeger sets of convex domains in the plane.

A convex body is said to be of constant width whenever the distance between two parallel supporting hyperplanes to this body is independent of the hyperplane. Balls are an obvious example but there are many others such as Reuleaux triangles in two dimensions and Meissner's tetrahedra in dimension three. Another difficult and challenging problem Thomas studied, in collaboration with Édouard Oudet is that of finding convex bodies of constant width and minimal volume. In two dimensions, it has been proven by Blaschke and Lebesgue that the solution is the Reuleaux triangle. In three dimensions, the problem is still open. Thomas and Édouard Oudet also found a method to construct a body of constant width in dimension n, starting from a given projection in dimension $n-1$ and gave a complete analytic parametrization of constant width bodies in dimension 3.

This special volume of the Journal of Convex Analysis dedicated to Thomas' memory reflects his many scientific interests in the calculus of variations, convex analysis, shape optimization and variational analysis.

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