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Linear Operators on Vector-Valued Function Spaces with Mackey Topologies

Let E be an ideal of L^0 over a σ-finite measure space (Ω, Σ, μ) and let E' be the Köthe dual of E. Let $(X, \|\cdot\|_X)$ be a real Banach space, and X^* the Banach dual of X. Let $E(X)$ be a subspace of the space $L^0(X)$ of μ-equivalence classes of all strongly Σ-measurable function $f : \Omega \to X$, and consisting of all those $f \in L^0(X)$ for which the scalar function $\widetilde{f}$, defined by $\widetilde{f}(\omega) = \|f(\omega)\|_X$ for $\omega \in \Omega$, belongs to E. Assume that a Banach space X is an Asplund space. It is shown that a subset C of $E'(X^*)$ is relatively $\sigma(E'(X^*), E(X))$-compact iff the set $\{\widetilde{g} : g \in E'(X^*)\}$ in E' is relatively $\sigma(E', E)$-compact. We consider the topology $\overline{\tau(E, E')}$ on $E(X)$ associated with the Mackey topology $\tau(E, E')$ on E. It is shown that $\overline{\tau(E, E')}$ is strongly Mackey topology; hence $\overline{\tau(E, E')}$ coincides with the Mackey topology $\tau(E(X), E'(X^*))$. Moreover, $E'(X^*)$ is $\sigma(E'(X^*), E(X))$-sequentially complete whenever E' is perfect. We examine the space $\mathcal{L}_\tau(E(X), Y)$ of all $(\tau(E(X), E'(X^*)), \|\cdot\|_Y)$-continuous linear operators from $E(X)$ to a Banach space $(Y, \|\cdot\|_Y)$, equipped with the weak operator topology (briefly WOT) and the strong operator topology (briefly SOT). It is shown that if E is perfect, then $\mathcal{L}_\tau(E(X), Y)$ is WOT-sequentially complete, and every SOT-compact subset of $\mathcal{L}_\tau(E(X), Y)$ is $(\tau(E(X), E'(X^*)), \|\cdot\|_Y)$-equicontinuous. Moreover, a Vitali-Hahn-Saks type theorem for $\mathcal{L}_\tau(E(X), Y)$ is obtained.

Keywords: Vector-valued function spaces, Mackey topologies, strongly Mackey topologies, weak compactness, Radon-Nikodym property, Asplund spaces, sequential completeness, convex compactness property, weak operator topology, strong operator topology, linear operator

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