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Hausdorff Dimension of Cut Loci of Generic Subspaces of Euclidean Spaces

Let F be a closed set of the Euclidean space $\mathbb{E}^d$, with $\emptyset \neq F \neq \mathbb{E}^d$ and $d \geq 2$. Let $\mathcal{N}$ be the set of centers of all open balls contained in $\mathbb{E}^d \setminus F$ which are maximal with respect to inclusion. We prove that the Hausdorff dimension $\dim_{\mathrm{H}}(\mathcal{N})$ of $\mathcal{N}$ equals d when F is, in the sense of Baire categories, a generic compact subset of $\mathbb{E}^d$, or when $\mathbb{E}^d \setminus F$ is the interior of a generic convex body of $\mathbb{E}^d$. If C is a generic convex body, we deduce that the set of all points of ∂C where the "upper curvature" of ∂C is positive and finite, is of Hausdorff dimension $d-1$. Let CurvCt be the set of centers of upper curvature of ∂C, and ω be any non empty open subset of $\mathbb{E}^d$. We also prove that $\dim_{\mathrm{H}}(\omega \cap \mathrm{CurvCt}) = d$. Let B be a generic compact subset of $\mathbb{E}^d$, or a generic convex body of $\mathbb{E}^d$. Let $\mathrm{a}\mathcal{N}$ be the set of centers of all closed balls containing B which are minimal with respect to inclusion. We also prove that $\dim_{\mathrm{H}}(\mathrm{a}\mathcal{N}) = d$. The proofs employ some of the ideas used in a previous paper of the author ["Dimension de Hausdorff de la nervure", Geom. Dedicata, 85 (2001) 217–235] to construct large cut loci in $\mathbb{E}^d$.

Keywords: Cut locus, skeleton, medial axis, Hausdorff dimension, critical value, curvature, convex body, farthest distance.

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