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On the Directions of Segments and r -Dimensional Balls on a Convex Surface

We prove that the set of directions of $(n - 2)$ -dimensional balls which are contained in the boundary ∂K of a convex body $K \subset \mathbb{R}^n$ but in no $(n - 1)$ -dimensional convex subset of ∂K is σ -1-rectifiable. We also show that there exists a close connection between smallness of the set of directions of line segments on ∂K and smallness of the set of tangent hyperplanes to the graph of a d. c. (delta-convex) function on \mathbb{R}^{n-2} . Using this connection, we construct $K \subset \mathbb{R}^3$ such that the set of directions of segments on ∂K cannot be covered by countably many simple Jordan arcs having half-tangents at all points. Also new results on directions of r -dimensional balls in ∂K parallel to a fixed linear subspace are proved.

Keywords: Segments and balls on the boundary of a convex body, Hausdorff measure, tangent hyperplane, d. c. function.

MSC: 52A20; 26B25