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Existence and Relaxation Theorems for Unbounded Differential Inclusions

We are interested in the existence of solutions of the differential inclusion

$$\dot{x} \in F(t,x)$$

on the given time interval, say $[0,1]$. Here F is a set-valued mapping from $[0,1] \times \mathbf{R}^n$ into $\mathbf{R}^n$ (we shall write $F : [0,1] \times \mathbf{R}^n \rightrightarrows \mathbf{R}^n$ in what follows) with closed values which will be assumed nonempty whenever necessary. The classical theorems of Filippov and Wazewski theorem uses, as the main assumption characterizing the dependence of F on x, the standard Lipschitz condition

$$h(F(t,x), F(t,x')) \leq k(t)\|x - x'\|,$$

where $h(P,Q)$ stands for the Hausdorff distance from P to Q. This condition, quite reasonable when F is bounded-valued, becomes unacceptably strong if the values of F can be unbounded. Meanwhile unboundedness of the values of the right-hand side set-valued mapping is a fairly natural property of differential inclusions which appear in optimal control problems, e.g. when we deal with a Mayer problem obtained as a result of reformulation of a problem with integral functional. The main purpose of this note is to provide an existence theorem with a weaker version of the Lipschitz condition which is "more acceptable" when the values of F are unbounded. This condition which could be characterized as a "global" version of Aubin's pseudo-Lipschitz property is very close to that introduced by P. D. Loewen and R. T. Rockafellar [SIAM J. Control Optimization 32 (1994) 442–470].

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